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Properties of Strange Quark Matter in the Context of Diquark Correlation

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Properties of strange quark matter (SQM) have been studied as diquark matter with diquark correlation. The dense SQM is under strong magnetic field due to electroweak interaction of quarks. Here it is suggested that u, d, s quarks get correlated to form diquark matter of different flavors. The diquarks are suggested to behave like quasiparticles under the influence of strong magnetic field inside the dense quark matter. In the current work diquarks are described as the composite fermions which behave like quasiparticle under the strong magneticfield in an analogy with the electrons behaving in strong magnetic field. The thermodynamic potential for SQM has been estimated and behavior is studied with respect to chemical potential. The equation of state and the velocity of sound through SQM are also studied. Some interesting observations are made. It is suggested that the composite fermion picture of diquark correlation lowers the binding energy of the system making the SQM a more stable state.

1. Introduction

The strange quark matter (SQM) is a new state of matter where u, d, and s quarks are in de-confined phase. Witten[1] has suggested that this new form of matter may be the true ground state of strong interaction. Due to the asymptotic freedom of strong interaction (QCD), there is a possibility of formation of SQM at high densities.[2,3] It has been suggested that the SQM is more stable than the nuclear matter due to the conversion of u, d quarks to strange quark by weak interaction at high densities and possible reduction of free energies due to the conversion of non-strange quarks to strange quarks. The SQM approximately consists of equal numbers of u, d, and s quarks which can be described as exotic matter with perfect density and stability. In recent years a number of works on SQM are in progress both in theoretical and experimental fronts. The core of dense stars, pulsars, neutron stars, and the presence of strange quark stars support the existence of SQM.[4–7] The study of SQM with quasi particle is one of the most interesting area of research. The masses of quarks vary with environment and it is suggested to be the function of density, momentum incorporating medium effect in the context of fermi model. Wen et al.[8] have studied SQM and strangelet considering medium dependent quark mass with respect to chemical potential and temperature. They have suggested that the quarks behave like quasi particle under the environmental effect. Cai et al.[9] have investigated strange quark stars in the frame work of quasi particle model. The equation of state of strange quark stars are obtained considering the effect of chemical equilibrium and charge neutrality whereas Wu et al.[10] have investigated the SQM in quasi particle model and energy of baryons as a function of chemical potential.

The properties of SQM in strong magnetic field are another interesting area of study. It has been suggested that the magnetic field inside SQM grows due to instability driven by electroweak interaction. Dvornikov[11] has made an interesting observation regarding the high magnetic effect in SQM which can exist in the core of HS and QS and hints at the possibility of magnetors. The diquark matter is also observed in abundance in stellar matter. Recently it has been suggested that an intermediate stage in the nuclear matter to ordinary quark matter in which nucleons are dissociated and majority of quarks remain localized in spin singlet pairs which are diquark states. The pairing of quarks is observed in high density QCD perturbation theory which happens via NJL mechanism.[12,13] Horvath et al.[14] have studied the role of diquarks in dense matter and have studied its astrophysical realization in the core of the neutron stars.

In the present work we have studied the SQM describing the state as system of diquarks. It is stated that a large scale magnetic field is created in SQM due to the electroweak interaction between quarks. We have suggested that under the influence of this strong magnetic field the quarks form correlated diquarks and behave as quasi particles in an analogy to electrons in high magnetic field which are described as composite fermions (CF). It is interesting to study how the binding energy of diquarks changes the state of SQM. We have studied the thermodynamic potential of the system with contribution from [uu], [ss], and [us] diquarks.

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The equation of states and the velocity of sound are also studied with diquark correlation.

2. Formulation

2.1. Composite Fermion Model for Diquarks

Recently we have suggested a composite fermion model for diquark.[15] It is known that in the presence of the magnetic field quantum particles behave like a quasi particle (composite fermions [CF]) incorporating the effect of the magnetic field.[16] The effective mass of the composite fermion is described as self energy and it depends on the gauge choice of Hamiltonian. The masses can be estimated in the similar way as that of effective mass of the Fermi liquid. It has been suggested the CF carries flux quanta of the field and cancels the external magnetic field forming a stable quasi particle system. The effective mass incorporates the interaction of the magnetic field and it is expressed in terms of a cut off parameter and Fermi momentum. A comprehensive work on composite fermions is done by Jain[16] where the properties of composite fermions and related studies on quantum Hall effect have been discussed. A number of works have been done on composite fermions associated with experimental evidence of CF.[17–19] In the context of diquark we have suggested that in the presence of QCD vacuum and chromo-electromagnetic field, diquark behaves like a composite fermion and as an independent entity like quasi particle. The effective mass of the diquark as composite fermion can be computed in a gauge invariant way. Chari et al.[20] have proposed effective mass of quasiparticle in gauge invariant way and expressed it in term of response function of the system in analogy with Landau's picture of quasiparticle of Fermi liquid where the dressing of particles or (holes) in the free Fermi gas due to interaction has been described. Starting from Hamiltonian of composite fermion with cut off Λ, the expression of quasi particle mass in a gauge invariant system can be obtained as[20]

$$\frac{1}{m^*} = \frac{1}{m}(1 + \frac{\Lambda^4}{2p_\mathrm{F}^4}) \tag{1}$$

where m^* is the effective mass of quasiparticle, m is the constituent particle mass. p_F is the Fermi momentum and Λ is a cut off parameter. Considering diquark behaving like CF in presence of chromo-magnetic field, the effective mass of diquark can be expressed as

$$\frac{1}{m_\mathrm{D}^*} = \frac{1}{m_{\mathrm{q}_1} + m_{\mathrm{q}_2}}(1 + \frac{\Lambda^4}{2p_\mathrm{F}^4}) \tag{2}$$

where m_D^* is the mass of the diquark, m_{q_1}, m_{q_2} are the constituent masses of the quark flavors constituting the diquark. The diquark masses have been estimated using Equation (2) with $m_\mathrm{u} = m_\mathrm{d} = 0.360$ GeV, and $m_\mathrm{s} = 0.540$ GeV, $\Lambda = 0.573$ GeV[21] and Fermi momentum p_F has been estimated using work of Bhattacharya et al.[22] The effective masses for diquarks are found as $m_\mathrm{ud} = 0.4848$ GeV, $m_\mathrm{us} = 0.6331$ GeV, $m_\mathrm{ss} = 0.7149$ GeV for spin singlet scalar diquarks. The effective mass of the [ss] diquark has been estimated with radius as 5 GeV $^{-1}$. We have used the computed effective masses of diquarks in our subsequent studies of properties of SQM.

3. Thermodynamic Potential

The quasiparticle contribution to the thermodynamic potential can be expressed as[8,9]

$$\Omega_i = \frac{d_i}{2\pi^2}\int_0^\infty \{\ln[1 + \mathrm{e}^{-(\epsilon_{i,p}-\mu_i)/T}] + \ln[1 + \mathrm{e}^{-(\epsilon_{i,p}+\mu_i)/T}]\}p^2\mathrm{d}p \tag{3}$$

where $\epsilon_{i,p} = \sqrt{p^2 + m_i^{*2}}$ and T is the temperature. Considering T= 0, we arrive at

$$\Omega(\mu_i, m^{*_i}) = -\frac{d_i}{48\pi^2}[|\mu_i|\sqrt{\mu_i^2 - m_i^{*2}}(2\mu_i^{*2} - 5m_i^{*2}) + 3m_i^{*4} ln\frac{|\mu_i| + \sqrt{\mu_i^2 - m_i^{*2}}}{m_i^*} \tag{4}$$

with degeneracy factor d_i for quarks,the total thermodynamic potential for SQM can be expressed as[9]

$$\Omega_i = \sum \Omega_i(\mu_i, m_i^*) + B(\mu_i) + B_0 \tag{5}$$

B(μ_i)can be expressed as

$$B(\mu_i) = -\frac{d_i a^2}{16\pi^4}[\sqrt{1-a^2} - a^2\ln\frac{1+\sqrt{1-a^2}}{a}]\mu_i^4 \tag{6}$$

a = g/($\sqrt{6}\pi$). The degeneracy factor d_i for quarks is 6 and g is the coupling constant. We have estimated thermodynamic potential for SQM with the input of our diquark masses evaluated in the framework of composite fermion model and plotted the variation of Ω with chemical potential μ for [uu], [us], and [ss] diquark contribution using Equation (5) where the SQM is described as diquark matter with different diquark correlations. The results are displayed in the **Figure 1**.

4. Equation of States of Diquark Matter and Velocity of Sound

Diquark matter is color antisymmetric. The equation of states of diquark matter has been suggested as[23]

$$P = \frac{\lambda}{2m_\mathrm{D}^4}\rho^2 \tag{7}$$

where P is the pressure, ρ is the density, m_D is the corresponding diquark mass, and λ dimensionless strong coupling constant. Expression for the velocity of sound through the medium can be written as

$$c_\mathrm{s}^2 = \frac{\mathrm{d}P}{\mathrm{d}\rho} = \frac{\lambda}{m_\mathrm{D}^4}\rho \tag{8}$$

We have studied the variation of pressure with density for [uu], [us], [ss] diquarks with our description of diquark as composite

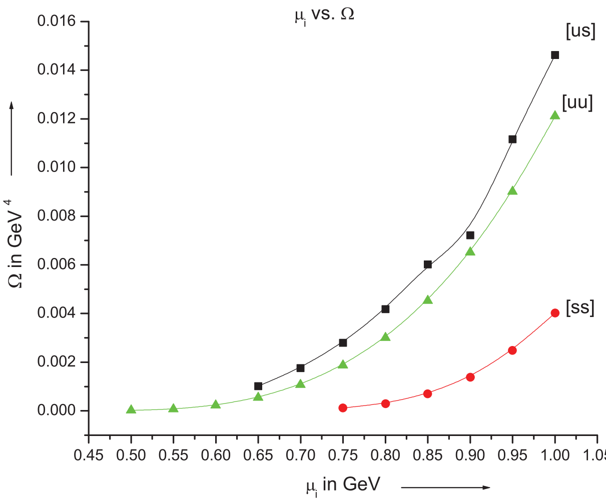

Figure 1. Variation of thermodynamic potential with chemical potential for [uu], [us], and [ss] diquark.

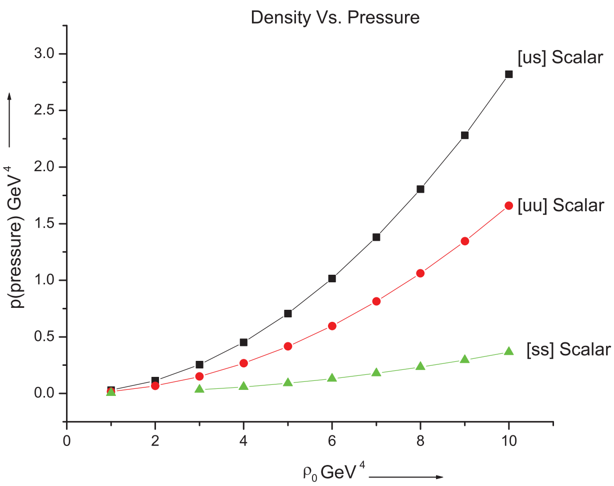

Figure 2. Variation of pressure with density for [uu], [us], and [ss].

fermion and displayed the result in **Figure 2**, upto $\rho = 10\rho_0$, where ρ_0 is the normal nuclear matter density. The velocity of sound through different diquark matter have been estimated and furnished in **Table 1** along with the conformal and causality limits.

5. Results and Discussions

In the current work we have investigated the properties of SQM with diquark correlation. The SQM is described as diquark matter. It has been suggested that the binding energies of diquarks affect the properties of SQM to a great extent. Contributions for different diquark correlations to thermodynamic potential energies have been estimated with quasi particle model of diquark suggested by us.[15] The variation of thermodynamic potential with chemical potential has been computed and are displayed in Figure 1. It shows the variation of thermodynamic potential with chemical potential for different diquarks. Ω is found to be monotonically increasing function of chemical potential. It is interesting to note that in each case the thermodynamic potential possesses a critical value after which the diquark correlation occurs. Cai et al.[9] have observed same type of behavior for baryon energy density with baryon chemical potential while investigating properties of strange quark stars with quasi particle model. Horvath et al.[14] have pointed out that there may be presence of diquark dominated region immediately above the deconfinement density. Bailin et al.[24] have studied the SQM considering diquark correlation as an intermediate stage of nuclear matter and described the correlation as high density QCD perturbation in NJL model. Diquark correlation has been studied by Bhattacharya et al.[15] considering the effect of choromoelectromagnetic nature of QCD vacuum in the context of composite fermion model. Stotami et al.[25] have pointed out that quark matter with strong magnetic field indicate the possibility of compact star whereas Noronha et al.[26] indicated the possibility of forming diquark Cooper pair participation in dense quark matter. They have argued that the mass of the strange quark should range 100 to 500 MeV in dense baryonic matter. It may be mentioned that the effect of [ud] and [ds] diquarks will be the same contribution as that of [uu] and [us] diquarks in the present formulation as u and d quarks masses are considered to be the same from SU(2) symmetry but they will be present for equilibrium condition of charge neutrality and color degree of freedom.

Table 1. Velocity of sound in different diquark matter, $\lambda = 0.117$, c-velocity of light.

Diquark	Velocity of sound c_s^2/c^2 $(\rho = \rho_0)$	Conformal limit of c_s	Causal limit c_s	Sound velocity c_s/c $(\rho = \rho_0)$
[uu]	0.260	$\approx 1.3\rho_0$	$3.86\rho_0$	0.510
[us]	0.444	$\approx 0.8\rho_0$	$2.25\rho_0$	0.816
[ss]	0.057	$6.6\rho_0$	$17.54\rho_0$	0.239

The equation of states and velocity of sound of SQM are studied. The value of the strong coupling constant is taken as $\lambda = 0.117$.[27] Figure 2 shows the variation of pressure with density. It is observed that at low density the equation of state is soft whereas the equation of state becomes hard as density increases. It may be pointed out that [ss] diquark matter shows very small variation of pressure with density indicating the property of incompressible liquid and shows a rise beyond $\rho = 8\rho_0$. The velocity of sound are estimated and furnished in Table 1 along with causality and conformal limits. It has been suggested that the breaking of causality limit and conformal limits indicates the instability region.[28,29] In the current work, it is observed that the [ss] diquark matter is stable even at density as high as $\approx 6.6\ \rho_0$ whereas for [uu] and [us] diquark matter, velocity of sound crosses the causality limit in the density zone $\approx 2\text{–}3\ \rho_0$. Recently Pegios et al.[30] have studied equation of states of dense quark matter studying the velocity of sound in the context of studying the tidal deformability. They have studied GW170817 data and GW190425 data and have pointed out that in dense quark matter speed of sound is lower than $\frac{c}{\sqrt{3}}$ upto a transition density $\rho = 1.6\ \rho_0$ and sound velocity equals velocity of light at $\rho = 1.8\ \rho_0$ whereas GW190425 event favors $\rho = \rho_0$ for lower speed of sound and $\rho = 1.2\ \rho_0$ for upper one. High speed of sound implies the strongly interacting system and in the present context it seems that [us] diquark matter is most strongly interacting system. It can be attributed to the high value of coupling constant. Moreover it is suggested that the strongly interacting systems of the quasiparticles may form a complete new state of matter called "unitary fermions" whose statistics is not exactly known. Speed of sound may be high in the bosonic state due to dropping in damping rate. As the diquarks are described as composite fermions which are quasi particle, we suggested that the high speed may be attributed to the strong correlation of the particles.

Current work focuses on the possibility of diquark correlation SQM. The magnetic field present in SQM initiates diquark correlation and in our study we have incorporated the effect through the composite fermion picture of quasi particles. The effective masses of the diquarks are found to be smaller than the constituent masses lowering the binding energies of the system which makes the system more stable. Linares et al.[31] have studied SQM matter with chromodielectric mode (CDM) with BCS quark pairing and investigated the color flavor locked phase. They have pointed out that the inclusion of negative free energy of diquark condensate plays an important role in stability of SQM whereas Schtler et al.[32] have observed that the medium reduces the overall binding energy of SQM. It has been suggested that the binding energy of diquarks affects the physical phenomenon vastly from nuclei to neutron stars. The cores of neutron and compact stars possess huge pressure which is able to deconfine neutrons and other hadronic constituents into their quark constituents. The magnetic field in SQM favors the diquark correlation. In this work we have studied the possibility of diquark formation in such dense matter promoted by the effect of high magnetic field present and is corroborated by composite fermion type of quasiparticle behavior of diquarks. Diquark is color flavor locked phase and contribution of diquark binding energy may have a considerable effect in manifestation of properties of SQM. Diquark condensates are superconducting phase with typical band gap of 100 Mev. Blaschke et al.[33] have studied diquark condensate with signal to color superconducting phase and studied corresponding EOS under condition of beta equilibrium and color neutrality. Our investigation has implications in the study of compact star matter as superconducting phase, pulsar, and astrophysics. The velocity of sound is a macroscopic parameter which is of great importance for studying the property of a system. It is pertinent to point out here that the diquarks behaving like composite fermion is a unique approach to study the SQM incorporating the effect of magnetic field. Present study will throw a light on our understanding of properties of SQM and dense quark stars. The bulk properties of SQM can also be studied with diquark correlation. Xiaoping et al.[34] have studied the viscosity of interacting SQM and observed that the viscosity is ten times greater than non interacting SQM. The exact nature of SQM is yet to be revealed. A new state of matter "unitary fermion" is recently studied for strongly interacting matter and may be a strong candidate for SQM. In our future studies we will investigate the bulk properties like viscosity, incompressibility, etc. considering interacting diquarks in the context of composite fermion model of diquark and superconducting phase phenomenon.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available in the supplementary material of this article.

Keywords

diquark, equation of states, sound velocity, strange quark matter, quasi particles

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